

Learning how to axiomatize through paperfolding

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Abstract

Obviously, paperfolding is no longer only art. In the last decades, it has been overwhelming to learn how much mathematics is waiting to be unfolded from a sheet of paper. Genuine origami mathematics, like flat foldability results or Haga's theorem, arises directly from the folding of paper and is not only visualized by origami, like folding of cubes and other polyhedra. Math educators became interested in origami as a useful tool to explain and teach mathematics as far back as in the 19th century (cf. Sundara Row, Friedrich Froebel). Today, paperfolding is often seen as a very beneficial tool in pupils' (math) education, although it is not easy to prove its effects (cf. [Golan and Jackson \(2009\)](#), [Arici and Aslan-Tutak \(2015\)](#), [Boakes \(2011\)](#) shed some light on it).

In German kindergartens and elementary schools there is a far back-reaching tradition in paperfolding. But only recently it is slowly getting its way into high schools and colleges. Despite the fact that a great part of nontrivial origami mathematics has been prepared for teaching at universities (for instance by [Hull \(2013\)](#)), there is little evidence of the presence of mathematical paperfolding in university education, apart from some few exceptions. Two reasons for this, among others, are: Only few non-folders know enough origami mathematics and if they do, they likely do not know how to fit origami into curricula. One way out of it is to train math educators in mathematical paperfolding, but it appears to be troublesome. Another way is to reach back to the source and teach origami at the university to prospective mathematics teachers in such a way that they learn some beautiful theories and results but also can later apply (downsized if necessary) their knowledge directly in classes.

In Germany, it is not easy to teach a new course at the university, which is not included in the curriculum, therefore such a course should be compatible and comparable with university education. But obviously, mathematical paper folding includes both mathematics *and* folding, so one must not reduce such a course to some high level algebra and field theory, but one should keep in mind that a lot of interesting school math hides, for instance, in the question how to exactly fold one fifth of a given segment; and what similarities and differences there are to a construction with euclidean tools.

As a result, the goal is to find a compromise in mathematical paperfolding between "folding cranes and calculating Galois extensions". We believe to see such a midway: We repeatedly

taught 1-fold-origami in one-term courses at University of Wuerzburg to prospective secondary level mathematics teachers and it turned out that topics like flat foldability, 1-fold-origami, solving equations and folding polyhedra can be organized in a connected, linear way (as opposed to some extent to Hull (2013)), where students fold and sort their findings, discover general patterns and basic folds, from which other foldings arise, culminating in finding all axioms of 1-fold-origami, the Huzita-Justin-axioms. From there we went on to axioms and their meaning in general and axioms of the euclidean plane as a special case, being the stage/playground for 1-fold-origami to take place, therefore a natural topic to be discussed whilst analyzing basic folds.

The idea of axiomatizing of 1-fold-origami prior to axiomatization of the euclidean plane follows the idea of training simple tasks before getting into trouble with more difficult ones. The main advantage of 1-fold-origami as a axiomatization tool could be seen in its concrete nature and handiness opposite to other more abstract systems, but we postpone a more detailed description as well as a discussion to the full paper.

One main goal of this PhD-research-project is to explore to what extent mathematical paper folding can help students to better understand axioms, axiomatization as a principle in general (cf. Yannotta (2013)) and one of the axiomatizations of the euclidean plane as a special case, being important for deeper understanding of constructions with ruler and compasses and school geometry. We conducted tests before and after every course to explore and better understand students' possible problems with axioms and used the ideas and insights to iteratively improve our courses.

In the full paper we explain the above ideas more elaborately, present details of the courses and give some results of the tests.

References

- Sevil Arici and Fatma Aslan-Tutak. The effect of origami-based instruction on spatial visualization, geometry achievement, and geometric reasoning. *International Journal of Science and Mathematics Education*, 13:179 – 200, 2015.
- Norma Boakes. Origami and spatial thinking of college-age students. In *Origami 5: Fifth international meeting of origami science, mathematics, and education (5OSME)*, pages 173–187. CRC Press, 2011.
- Miri Golan and Paul Jackson. Origametria: A program to teach geometry and to develop learning skills using the art of origami. In *Origami 4: Fourth International Meeting of Origami Science, Mathematics, and Education*, pages 459–469, 2009.
- Thomas Hull. *Project origami: activities for exploring mathematics*. CRC Press, 2013.
- Mark Yannotta. Students axiomatizing in a classroom setting. In *Proceedings of the 16th Annual Conference on Research in Undergraduate Mathematics Education*, volume 2, pages 290–297, 2013.